## Section III

**6.** (a) Show that $T : R^3 \to R^3$ defined by $T(x, y, z) = (x \cos\theta - y \sin\theta, x \sin\theta + y \cos\theta, z)$ is non-singular for all values of $\theta$. **2½**

(b) Find the matrix representing the transformation $T : R^2 \to R^3$ defined by $T(x, y) = (x + y, 2x - y, 7y)$ relative to the standard basis of $R^2$ and $R^3$. **2½**

**7.** (a) Five the co-ordinates of vector (1, 1, 1) relative to basis (1, 1, 2), (2, 2, 1), (1, 2, 2). **2**

(b) For a linear operator $T : R^3 \to R^3$, find the eigen values and basis for eigen space, when $T(x, y, z) = (x + y + z, 2y + z, 2y + 3z)$. **3**





# 32506

B.A. & Hons. (Subsidiary)

EXAMINATION, 2025

(Sixth Semester)

(Regular & Re-appear)

MATHEMATICS

BM-362

Linear Algebra

*Time : 3 Hours*] [*Maximum Marks :* 26

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Before answering the question-paper, candidates must ensure that they have been supplied with correct and complete question-paper. No complaint, in this regard will be entertained after the examination.

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**Note** : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. **1** is compulsory.

**(Compulsory Question)**

1. (a) Define internal and external binary operations. **1**

(b) Show that the set {(1, 2, 3), (1, 0, 0), (0, 2, 3)} in $R^3$ is linearly dependent. **1**

(c) Define linear transformation. **1**

(d) State Sylvester's Law. **1**

(e) Define A(W)-Annihilator of W. **1**

(f) Define norm of a vector. **1**

## Section I

2. (a) Show by an example, the union of two subspaces of a vector space $V_3(R)$ may not be a subspace of $V_3(R)$. **2½**

(b) For what value of $k$ (if any) the vector $v = (1, -2, k)$ can be expressed as a linear combination of vectors $v_1 = (3, 0, -2)$ and $v_2 = (2, -1, -5)$ in $R^3(R)$. **2½**

3. (a) Show that the vectors (2, –1, 0), (3, 5, 1), (1, 1, 2) form a basis of $R^3$. **2½**



(b) If W is a subspace of $v_3(R)$ generated by {(1, 0, 0), (1, 1, 0), find $v/w$ and its basis. **2½**

## Section II

4. (a) Show that the transformation $T : R^3 \rightarrow R^2$ defined by $T(x_1, x_2, x_3) = (x_1, x_2)$ is a linear transformation and is onto but not one-one. **2½**

(b) Find linear transformation $T : R^3 \rightarrow R^3$ whose range space is spanned by the vectors (1, 2, 3), (4, 5, 6). **2½**

5. (a) Prove that the images of linearly independent vectors under a linear transformation need not be linearly independent. **2½**

(b) If W is any subset of a vector space V(F), then prove that A(W) is a subspace of V*. **2½**

## Section IV

**8.** (a) State and prove triangle in equality. **2½**

(b) Let W be a non-empty subset of an inner product space V(F). Then prove that $W^{\perp}$ is a subspace of V(F). **2½**

**9.** (a) Let S be a subset of an inner product space V. Then show that $S^{\perp} = S^{\perp\perp\perp}$. **2½**

(b) Let T be a linear operator on a unitary space V. Then T is normal off :

$\|T^*(u)\| = \|T(u)\|$ for all $u \in V$. **2½**

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